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An Asymptotic Approach for the Collatz Conjecture

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Abstract

It is well known that the following Collatz Conjecture is one of the unsolved problems in mathematics.

Collatz Conjecture : For any positive integer $n > 1$, the following recursive algorithm will come to 1 by a finite number of steps.

f_1) If n is an even number then $\frac{n}{2} \to n$,

f_2) If n is an odd number then $\frac{3n+1}{2} \to n$.

Supercomputer simulation has confirmed that the Collatz conjecture is correct for all positive integers until 2.95×10^{20}. Recently, Terence Tao proved that almost all orbits of the Collatz map attain almost bounded values. It is to say that the Collatz conjecture is almost correct for almost all positive integers.

This paper proposes an asymptotic approach for the Collatz Conjecture by the mathematical induction and probability theory. The result of this paper means that the Collatz conjecture is correct for almost all positive integers.

Keywords : Collatz Conjecture, asymptotic approach, probability, mathematical induction.

1 Introduction

For any positive integer $n > 1$, we consider a recursive algorithm by repeating the following two steps f_1) and f_2).

f_1) If n is an even number then $\frac{n}{2} \to n$,

f_2) If n is an odd number then $\frac{3n+1}{2} \to n$.

For example, let $n = 7$, this algorithm generates the following sequence and terminates to 1.

$$7 \to 11 \to 17 \to 26 \to 13 \to 20 \to 10 \to 5 \to 8 \to 4 \to 2 \to 1,$$

$$f_1 f_1 f_1 f_2 f_1 f_1 f_2 f_1 f_2 f_2 f_2(7) = 1.$$

In 1937, German mathematician Lothar Collatz shown a famous conjecture : For any positive integer $n > 1$, the above recursive algorithm always terminates to 1 by a finite number of steps [1][2]. It is also known as the $3n + 1$ problem, the Ulam conjecture, Kakutani's problem, the Thwaites conjecture, Hasse's algorithm, or the Syracuse problem [2]-[4]. It seems that the Collatz conjecture is still an unsolved problem up to now. Based on the supercomputer