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Polytope Typology A: The separation of facial polytopes in the morphology of the regular and semi-regular polyhedra and tessellations

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Abstract

Inspired by Critchlow [1], and Grünbaum and Shephard [2], previous work has proposed an integral 2.5D cubic schema of the regular and semi-regular polyhedra and polygonal tessellations of the plane for each class of symmetry. This schema is differentiated into an upper and lower layer of 4 polytopes each, and characterized by corresponding pairs of upper and lower polytopes [3]. The motif of paired two-step sequences of first alternating separation and morphological transformation of faces, and second morphological transformation and separation of faces is explored, which in 2D consideration of the 2.5D schema are disposed about the vertical axis, as characterized by the correspondence between the *PPs* of the lower and upper squares (rhombi).

Developing earlier sustained research [3−11], this paper addresses a deeper typology of morphological transformation of the primary polytopes, involving the separation of one gendered set of the negative (−ve), neutral (ntrl), or positive (+ve) facial polytopes along the Y, Z, and X axes of the cubic schema. While one set of faces separates, the other two sets morph or project through null→regular or quasi-regular→double facial levels (0→α|β→2) of the rhombic schema or its reflection. Each facial set only separates once, faces separating by d=0→1. The cubic schema exhibits significant three-fold symmetry by gender. The separation of faces schema adequately describes the morphology of the three classes of regular and semi-regular polyhedra of {2,3,3}, {2,3,4}, and {2,3,5} symmetry, and the two classes of polygonal tessellations of {2,3,6} and {2,4,4} symmetry.

Key words: morphology, polyhedra, separation of faces, tessellations

1. Class II and generic pairing of polyhedra by the separation of faces

Figure 1 of the 2.5D schema shows that the pairings of polyhedra within any one class can be characterized by the separation of one set of the negative, neutral, or positive surface polytopes on the (left to right) Y, Z, or X axes, respectively. Three significant kinds of pairings of *PP*s are evident in the 2.5D schema, one for each orthogonal axis. These are described for Class II of {2,3,4} symmetry, which is characterized by the −ve, neutral, and +ve axes of the class, and thus of each of its individual polytopes, being the √1, √2, and √3 (100, 110, 111) axes, respectively, of the cube.

In this class of 3D polyhedra, the +ve and −ve polar polytopes assume different (dual) forms; this contrasts with Class I also of 3D polyhedra, in which the two polar polytopes take the same tetrahedral form, though in alternative orientation, or Class V of 2D polygons, in which both polar polytopes assume the same form of the square, but in different location. The symmetry axes in the other classes are not in general orthogonal; meanwhile, Class II precisely consists of the *PP*s of the Class III honeycombs. corresponding to the primary components of the Class III honeycomb periodic all-space-filling arrays. Later comparison of Classes II and IV illustrates the differences between 3D polyhedral and 2D polygonal form, while considering classes with different polar polytopes, as opposed to having the same, though reoriented (3D) or relocated (2D) form. The beautiful integrity of interrelationship is clearly revealed in Fig. 1: