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Polytope Typology E: Detailed Separation and Progression of the Polyhedra in the Class III All-space-filling Periodic Honeycombs

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Abstract

This paper develops a companion paper intended to be read in conjunction with it, the terms used in this paper being defined as in that paper [1]. Herein, I investigate the role of the separation (SoP) and progression (PoP) of the Primary Polyhedra (PP) in the all-space-filling periodic arrays of polyhedra (the honeycombs), and their analogous morphology to the separation (SoF) and progression of faces (PoF) in the polyhedra (PP to PP within their class, and generically across classes). In previous research, I classify the honeycombs into three classes by symmetry. Here, I address Class III, and find 10 distinct forms (16 allowing reflections) in 3 clusters (Primary, Secondary, Tertiary) that are pervaded by the SoP and the PoP of two PPs of Class II of Polyhedra that are √3 compatible (in 4 cases being self-compatible), and are organized on that basis, where each honeycomb consists of two arrays of primary forms, one of which separates from contracted (PP_1 contiguous) to expanded (PP_1 separated), while the other progresses (morphs $PP_2 \rightarrow PP_3$) while remaining contracted or expanded, where the overall honeycomb can be contracted (both PP_1 and PP_2), hybrid (PP_1/PP_2 contracted and PP_2/PP_1 expanded), or fully expanded (both PP_1 and PP_2). Subsequent papers will address Class II of 4 distinct honeycombs, which may also be organized in somewhat similar manner, and Class I of one distinct honeycomb in contracted form, to clarify the morphology of the honeycombs.

Keywords: morphology, separation of polyhedra, classification, order, harmony

Introduction

Each of the Classes I–III of polyhedra and Classes 4 & 5 of tessellations (2D tilings) that I have previously advanced contains 8 members that show equivalent behavior across the classes: the simplex Vertical Polytope (VP); the 2 Polar polyTopes ($PT^{+/-}$) & the Quasi-regular (QR)); the 2 Truncated Polytopes ($TP^{+/-}$) & the Small Rhombic QR (SR or $SRQR$); and the Great Rhombic QR (GR or $GRQR$). Class II of the polyhedra consists of VP_{II}; Octahedron OH, Cube CB, & Cuboctahedron CO; truncated Octahedron TO, Truncated Cube TC, & Small Rhombic Cuboctahedron $SRCO$; and Great Rhombic Cuboctahedron $GRCO$ [2]. These 8 PPs characterize the Class III honeycombs [3].

Faces (F) of polyhedra can be considered positive +ve, neutral (ntrl), or negative (−ve), and are generalized to surface polytopes to include vertices (VT) and edges (EG), some of which are orthogonal to symmetry axes and regarded as principial, while others are incidental and can be disregarded, such as the VTs and EGs of the SR and GR. TO and TC have mixed principial and pragmatic EGs (the former NEs; the later those around the √3 axis HX or TR face, respectively) [4].

Individual honeycombs can readily be differentiated into two arrays of PPs that are assigned +ve or −ve polarity, and their mediating PTs, which are termed the Neutral polyTopes (NT); and when their array is contracted can be simply the VT, EG, or F between adjoining PPs, termed Neutral VerteX (NX), Neutral Edge (NE), or Neutral Face (NF), respectively; or when expanded, their prismatic developments of unit height the verteX Prism (XP), Edge Prism (EP), or Face Prism (FP), respectively. Faces also include the Triangle (TR), Square (SQ), Rhomb (RM; simply the rotated square), Hexagon (HX), and Octagon (OG); but if neutral, the NX, NE, NS, NR, and NO, etc. Class III Face Prisms are the XP, Square Prism (SP), Rhombic Prism (RP; rotated SP or rotated CB), and Octagonal Prism (OP).